ChAv3

ex1 $105+110+\dots+1000 =$

$= (\underbrace{5+10+\dots+1000}_{200\text{ termes}}) - (\underbrace{5+10+\dots+100}_{20\text{ termes}})$ progr. arithm de raison 5

$= \frac{200}{2}(2\cdot5+199\cdot5) - \frac{20}{2}(2\cdot5+19\cdot5) = 99450$

ou $105+110+\dots+1000 = 5\cdot(21+22+\dots+200)$

$= 5[(1+2+\dots+200)-(1+2+\dots+20)]$

$= 5[\frac{200\cdot201}{2} - \frac{20\cdot21}{2}] = 99450$

ex2 $7+n_1+\dots+n_8+61 = \frac{10}{2}(7+61) = 340$

avec $n_1 = 7+r$
$n_2 = 7+2r$
$\vdots$
$n_8 = 7+8\cdot r$

$n_1+\dots+n_8 = 56+r(1+2+\dots+8) = 56+r(\frac{8\cdot9}{2}) = 56+36r$

donc $56+36r = 340-(7+61)$
$36r = 216$
$r = 6$

[rem: on peut aussi faire $\frac{61-7}{9} = 6$ pour trouver directement la raison!]

donc : 13-19-25-31-37-43-49-55

ex4 a) $2500\cdot(1+0{,}03)^{64} \simeq 16577{,}62$.-

b) $2500(1{,}03)^n = 5234{,}45 \Leftrightarrow 1{,}03^n = \frac{5234{,}45}{2500}$

$\Leftrightarrow n\log(1{,}03) = \log(\dots) \Leftrightarrow n = \frac{\log(\frac{5234{,}45}{2500})}{\log(1{,}03)} \simeq 25$

mort en 2024

ex3 $t_1 = 8,\ t_2 = 8r,\ t_3 = 8r^2 = 18$: $8r^2 = 18$
$r^2 = 9/4$
$r = \pm 3/2$

donc $t_{10} = 8\cdot r^{10-1} = 8(\pm\frac{3}{2})^9 \simeq \pm 307{,}5$

ex 5

$$S_n = 1 + 11 + 111 + 1111 + \dots + \underbrace{11\dots1}_{n \text{ termes}}$$

$$9S_n = 9 + 99 + \dots + \underbrace{99\dots9}_{n \text{ termes}}$$

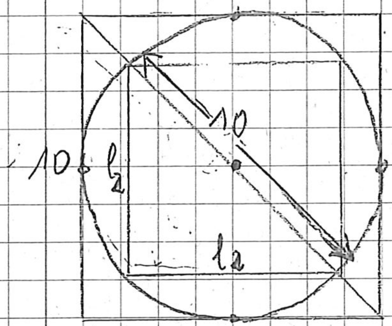
$$= (10-1) + (100-1) + \dots + (\underbrace{100\dots0}_{(n+1) \text{ termes}} - 1)$$

$$= \underbrace{10 + 100 + \dots + \underbrace{10\dots0}_{(n+1) \text{ termes}}}_{\text{progression géom } r = 10} - (\underbrace{1 + 1 + \dots + 1}_{n \text{ fois}})$$

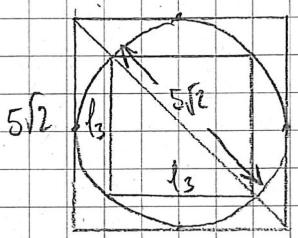
$$= 10 \cdot \frac{10^n - 1}{10 - 1} - n = 10 \, \frac{10^n - 1}{9} - n$$

donc $S_n = 10 \cdot \dfrac{10^n - 1}{81} - \dfrac{n}{9}$

ex 6



- $A_1 = 10^2 = 100$
  $l_1 = 10$
- $2 l_2^2 = 10^2$
  $l_2 = \sqrt{50} = 5\sqrt{2}$
  $A_2 = 25 \cdot 2 = 50$



- $2 l_3^2 = (5\sqrt{2})^2$
  $l_3 = \sqrt{25} = 5$
  $A_3 = 5^2 = 25$

etc $A_{n+1} = \dfrac{A_n}{2}$

prog géom de raison $r = \frac{1}{2}$

donc $A_1 + A_2 + \dots + A_n$

$$S_n = 100 \left(\frac{\left(\frac{1}{2}\right)^n - 1}{\frac{1}{2} - 1} \right)$$

$$\lim_{n \to \infty} S_n = 100 \left(\frac{0 - 1}{-\frac{1}{2}} \right) = 200 \text{ cm}^2$$

ex 7

$$S_n = 1 + \frac{3}{2^2} + \frac{5}{2^4} + \frac{7}{2^6} + \frac{9}{2^8} + \dots + \frac{2n+1}{2^{2n}}$$

$$\frac{1}{4} S_n = \frac{1}{4} + \frac{3}{2^4} + \frac{5}{2^6} + \frac{7}{2^8} + \dots + \frac{2n+1}{2^{2n+1}}$$

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$$S_n - \frac{1}{4} S_n = \frac{3}{4} + \frac{3}{4} + \underbrace{\frac{2}{2^4} + \frac{2}{2^6} + \dots + \frac{2}{2^{2n}}}_{r = 1/4} - \frac{2n+1}{2^{2n+1}}$$

$$\Leftrightarrow \frac{3}{4} S_n = \frac{3}{2} + \frac{2}{2^4}\left(\frac{(1/4)^{n-4} - 1}{1/4 - 1}\right) - \frac{2n+1}{2^{2n+1}}$$

$$\Leftrightarrow S_n = \frac{4}{\cancel{3}} \cdot \frac{\cancel{3}}{2} + \frac{4}{3\,8}\left[\frac{(1/4)^n - 1}{1/4 - 1}\right] - \frac{2n+1}{2^{2n+1}}$$

$$= 2 + \frac{1}{6}\left(\frac{(1/4)^n - 1}{-3/4}\right) - \frac{2n+1}{2^{2n+1}}$$

$$\lim_{\infty} S_n = 2 + \frac{1}{\cancel{6}_3} \cdot \frac{\cancel{4}^2}{3} - 0 = 2 + \frac{2}{9} = \frac{20}{9}$$

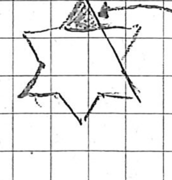
ex 8

a)

	# côtés	Périmètre
étape 0 :	3	c
1 :	$3 \cdot 4$	$3 \cdot 4 \cdot \frac{c}{3} = 3c\left(\frac{4}{3}\right)$
2 :	$(3 \cdot 4) \cdot 4 = 3 \cdot 4^2$	$3 \cdot 4^2 \cdot \frac{c}{3^2} = 3c\left(\frac{4}{3}\right)^2$
3 :	$(3 \cdot 4^2) \cdot 4 = 3 \cdot 4^3$	$3 \cdot 4^3 \cdot \frac{c}{3^3} = 3c\left(\frac{4}{3}\right)^3$
$\vdots$	$\vdots$	$\vdots$
k :	$3 \cdot 4^k$	$3 \cdot 4^k \cdot \frac{c}{3^k} = 3c\left(\frac{4}{3}\right)^k$

$$\lim_{\infty} P_n = 3c \cdot \infty = +\infty$$

b) étape 0 : $\frac{c \cdot h}{2}$ où $h^2 + \left(\frac{c}{2}\right)^2 = c^2 \Leftrightarrow h^2 = \frac{3c^2}{4} \Leftrightarrow h = \frac{\sqrt{3}\,c}{2}$: $A_0 = \frac{\sqrt{3}\,c^2}{4}$

1 : aire  $\frac{\sqrt{3}(c/3)^2}{4}$ $\Longrightarrow$ on en ajoute 3 : $3 \cdot \frac{\sqrt{3}(c/3)^2}{4}$

$$A_1 = \frac{\sqrt{3}\,c^2}{4} + \frac{3\sqrt{3}(c/3)^2}{4}$$

2 : on en ajoute $3 \cdot 4$ d'aires $\frac{\sqrt{3}(c/3^2)^2}{4}$; $A_2 = \frac{\sqrt{3}\,c^2}{4} + \frac{3\sqrt{3}(c/3)^2}{4} + \frac{3 \cdot 4\sqrt{3}(c/3^2)^2}{4}$

3 : " " " $3 \cdot 4^2$ " $\frac{\sqrt{3}(c/3^3)^2}{4}$

$\vdots$

$$A_3 = \frac{\sqrt{3}\,c^2}{4} + \frac{3\sqrt{3}(c/3)^2}{4} + \frac{3 \cdot 4\sqrt{3}(c/3^2)^2}{4} + \frac{3 \cdot 4^2 \cdot \sqrt{3}(c/3^3)^2}{4}$$

ex 8 (suite) étape k : on en ajoute $3 \cdot 4^{k-1}$ d'aires $\dfrac{\sqrt{3}\left(c/3^k\right)^2}{4}$

$$A_k = \frac{\sqrt{3}c^2}{4} + \frac{3\sqrt{3}\left(c/3\right)^2}{4} + \frac{3 \cdot 4\sqrt{3}\left(c/3^2\right)^2}{4} + \frac{3 \cdot 4^2\sqrt{3}\left(c/3^3\right)^2}{4} + \dots + \frac{3 \cdot 4^{k-1}\sqrt{3}\left(c/3^k\right)^2}{4}$$

$$= \frac{\sqrt{3}c^2}{4}\left[1 + 3 \cdot \frac{1}{3^2} + 3 \cdot 4 \cdot \frac{1}{3^4} + 3 \cdot 4^2 \cdot \frac{1}{3^6} + \dots + 3 \cdot 4^{k-1} \cdot \frac{1}{3^{2k}}\right]$$

$$= \frac{\sqrt{3}c^2}{4}\left[1 + \frac{1}{3} + \left(3 \cdot 2^2 \cdot \frac{1}{3^4} + 3 \cdot 2^4 \cdot \frac{1}{3^6} + \dots + 3 \cdot 2^{2k-2} \cdot \frac{1}{3^{2k}}\right)\right]$$

$$= \frac{\sqrt{3}c^2}{4}\left[\frac{4}{3} + \frac{1}{3}\left(\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + \dots + \left(\frac{2}{3}\right)^{2(k-1)}\right)\right]$$

$$= \frac{\sqrt{3}c^2}{4}\left[\frac{4}{3} + \frac{1}{3}\left(\left(\frac{2}{3}\right)^2\left[\frac{\left[\left(\frac{2}{3}\right)^{2}\right]^{k} - 1}{\left(\frac{2}{3}\right)^2 - 1}\right]\right)\right]$$

$$\lim_{\infty} A_k = \frac{\sqrt{3}c^2}{4}\left[\frac{4}{3} + \frac{1}{3} \cdot \frac{4}{\cancel{9}}\left[\frac{-1}{-\frac{5}{\cancel{9}}}\right]\right)\right] = \frac{\sqrt{3}c^2}{12}\left[\frac{4}{3} + \frac{4}{\cancel{27}_{3}} \cdot \frac{\cancel{9}}{5}\right]$$

$$= \frac{\sqrt{3}c^2}{\cancel{4}} \frac{\cancel{4}}{3}\left[1 + \frac{1}{5}\right] = \frac{\sqrt{3}c^2}{3}\left[\frac{6}{5}\right] = \frac{2\sqrt{3}c^2}{5}$$